

Q.3.  $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}$

char poly.  $\det(A - \lambda I)$   
 char eqn.  $\det(A - \lambda I) = 0$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 2 & -1 \\ 2 & 3-\lambda & -2 \\ -1 & -2 & -\lambda \end{vmatrix}$$

$$\lambda^3 - 3\lambda^2 - 9\lambda - 5 = 0$$

$$5 = \pm 1, \pm 5$$

$$(\lambda + 1)(\lambda^2 - 4\lambda - 5) = 0$$

$$(\lambda + 1)(\lambda + 1)(\lambda - 5) = 0$$

$$\lambda = -1, -1, 5$$

$$\lambda_1 \quad \lambda_2 \quad \lambda_3$$

$$(A - \lambda_i I) X_i = 0$$

$$(A + I) X_1 = 0$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A + I : b) = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & -2 & 0 \\ -1 & -2 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_2 = k_1 \quad x_3 = k_2 \quad x_1 + 2x_2 - x_3 = 0$$

$$x_1 = k_2 - 2k_1$$

$$X_{12} = \begin{bmatrix} k_2 - 2k_1 \\ k_1 \\ k_2 \end{bmatrix}$$

$$(k_1, k_2) = (1, 0) \quad (k_1, k_2) = (0, 1)$$

$$X_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$(A - 5I) X_3 = 0$$

$$(A - 5I : b) = \begin{bmatrix} -5 & 2 & -1 & 0 \\ 2 & -2 & -2 & 0 \\ -1 & -2 & -5 & 0 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} 2 & -2 & -2 & 0 \\ -1 & -2 & -5 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 \\ 2 & -2 & -2 & 0 \\ -5 & 2 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & -6 & -12 & 0 \\ 0 & 12 & 24 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & -6 & -12 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \textcircled{1} & 2 & 5 & 0 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_3 = K, \quad x_2 + 2x_3 = 0 \Rightarrow x_2 = -2K$$

$$x_1 + 2x_2 + 5x_3 = 0 \Rightarrow x_1 = -K$$

$$X_3 = \begin{bmatrix} -K \\ -2K \\ K \end{bmatrix} = K \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad P^{-1} = \frac{1}{6} \begin{bmatrix} -2 & 2 & 2 \\ 1 & 2 & 5 \\ -1 & -2 & 1 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{6} \begin{bmatrix} -2 & 2 & 2 \\ 1 & 2 & 5 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} -2 & 2 & 2 \\ 1 & 2 & 5 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -5 \\ -1 & 0 & -10 \\ 0 & -1 & 5 \end{bmatrix}$$

$$\rightarrow \frac{1}{6} \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 30 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = D$$

Def. Given a square matrix  $A$  if find  $P$  &  $P^{-1}$  s.t.  $P^{-1}AP = D$  then  $A$  is diagonalizable.

1) If  $\lambda_1, \lambda_2, \dots, \lambda_n$  are distinct then  $A$  is diag

2)  $A$  is diag. if  $AM > GM$  for "each"  $\lambda$ .

$AM > NO.$  of times an eigen value repeats.

$GM$  of  $\lambda_i = \text{Multiplicity of } (A - \lambda_i I)$

$$\Rightarrow -1 \quad AM > 2 \quad \quad \quad 5AM > 1$$

-1  $GM > \text{rank}(A+I) = \text{NO. of zero rows after REF.}$

$$(A+I) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad -1 \quad GM = 2$$

$$5GM > N(A-5I)$$

$$(A-5I) = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \quad 5GM = 1$$

5) ii) iii)

$$3) \det = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_n$$

$$\text{Trace} = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

$$A) \quad A \rightarrow \lambda_1, \lambda_2, \dots, \lambda_n \quad A^n \rightarrow \lambda_1^n, \lambda_2^n, \dots, \lambda_n^n$$

$$A^{-1} \rightarrow \frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$$

$$78) \quad \lambda_1 \lambda_2 = 5 \quad \text{Find e.v.}$$

$$\lambda_1 + \lambda_2 = 4$$

$$79) \quad A \text{ char poly. } \lambda(\lambda-1)(\lambda+2) = 0$$

$$A^2 \text{ char. poly. ??} \quad \lambda = 0, 1, -2$$

$$A^2 = 0, 1, 4$$

$$A^2 \text{ char poly. } (\lambda-0)(\lambda-1)(\lambda-4)$$

$$Q := \left\{ \underset{x_1}{(1, 0, 0)}, \underset{x_2}{(1, 1, 0)}, \underset{x_3}{(1, 1, 1)} \right\}$$

$$\{v_1, v_2, v_3\}$$